

# CHAPTER SIX

## BEATS, DOPPLER EFFECT AND RESONANCE

### **Beat:**

- When two sound notes of nearly equal frequency, travel in a sinusoidal manner along a straight line, in the same direction in a medium, there is a periodic rise and fall in intensity (i.e. loudness), which is referred to as beat.
- It is a well known fact that if two notes of equal frequency are sounded together, a periodic rise and fall in intensity can be heard, and this is known as the phenomenon of beats.
- The number of intense sound heard per second is called the beat frequency ( $f$ ), and is numerically equal to the difference between the frequencies of the two interfering waves (i.e.  $f = f_1 - f_2$ ).
- In other words, the beat frequency is always equal to the difference of the two equal frequencies.
- The frequency of beat  $f$  is equal to the difference of the frequencies i.e.  $f = f_1 - f_2$ .  
The resultant displacement of any point in space continually changes with time (interference in time), alternatively becoming maximum and minimum as the two wave trains get in and out of phase with each other, at regular intervals of time.

### **Uses of beats:**

- (1) It is used to measure the unknown frequency of a note.
- (2) It is used to tune an instrument to a given note.

#### **Use of beat to measure the unknown frequency of a note:**

- In order to measure the unknown frequency,  $f_1$ , of a note, a note of known frequency,  $f_2$  is used to provide beats with the unknown note, and the frequency  $f$  of the beats is obtained by counting the number made in a given time.
- Since  $f$  is the difference between  $f_2$  and  $f_1$ , it follows that  $f_1 = f_2 - f$ , or  $f_1 = f_2 + f$  i.e.  $f_1 = f_2 \pm f$ .
- In order to determine which value of  $f_1$  is correct, the end of the tuning – fork pong (i.e. the fork of known frequency) is loaded with a piece of plasticine, which diminishes its frequency a little.

- The forks are now used to produce beats again, and if the number of beats is reduced then  $f_1$  was below  $f_2$  i.e.  $f_1 = f_2 - f$ .
- On the other hand, if the number of beats increases, then  $f_1$  was above  $f_2$  i.e.  $f_1 = f_2 + f$ .

### **Tuning an instrument to a given note:**

- The frequency or pitch of the instrument is adjusted, so that its frequency approaches very close to that of a sounding tuning fork.
- As the instrument's note approaches a given note (e.g. the frequency of the tuning fork), beats are heard and the instrument can be regarded as "tuned".

(Q1)(a) How will you demonstrate in the laboratory the phenomenon of beat?

(b) Describe one practical application of beats.

Soln:

(a) The phenomenon of beat can be demonstrated by taking two tuning forks, of the same pitch, mounting them on sound boxes, and loading one with an appropriate material such as sealing wax or plasticine, to reduce its frequency slightly.

The two tuning forks are then struck and beats can be heard distinctly.

(b) - The phenomenon of beat can be used to measure the unknown frequency,  $f$ , of a note.

- For this purpose, a note of known frequency,  $f_2$  is used to provide beats with the unknown note, and the frequency  $f$  of the beats is obtained by counting the number made in a given time.
- Since  $f$  is the difference between  $f_2$  and  $f_1$ , it follows that  $f_1 = f_2 - f$  or  $f_1 = f_2 + f$ .
- To decide which value of  $f$  is correct, the end of the tuning – fork prong is loaded with plasticine and the two notes are sounded again.
- If the beats are increased then the frequency of the note must be  $f_1 = f_2 - f$ .
- If the beats are decreases, then the frequency is  $f_1 = f_2 + f$ .

(Q2) A school bell has a frequency of 126Hz, and there is a periodic rise and fall of loudness of this frequency six times a second. At which two frequencies are parts of the school bell ringing.

Soln:

Let  $f_1$  and  $f_2$  = the frequencies of the bell.

The beat frequency  $6 = f_1 - f_2$ .....Eqn (1).

The resultant frequency  $126 = \frac{f_1 + f_2}{2} \Rightarrow 2 \times 126$

$= f_1 + f_2 \Rightarrow 252 = f_1 + f_2$  .....Eqn (2)

Solving equations (1) and (2) simultaneously  $\Rightarrow f_1 = 129\text{Hz}$  and  $f_2 = 123\text{Hz}$ .

**Doppler effect:** This occurs whenever a source of sound or light moves relative to an observer, leading to a change in the frequency of the wave motion. For example, the whistle of a train or a jet plane, appears to increase in pitch as it approaches a stationary observer, but as the moving object (train or jet) passes the observer, the pitch changes and becomes lower.

**Calculation of apparent frequency:**

Let  $f^1$  = apparent frequency.

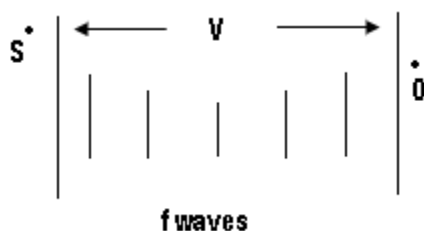
$V$  = Velocity of sound in air.

$U_s$  = velocity of the source of sound  $S$ .

$U_o$  = the velocity of the observer  $O$ .

$f$  = frequency of the source.

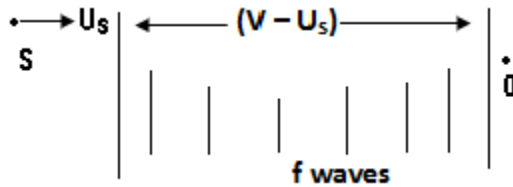
(1)



**Source stationary:**

- If the source  $S$  is stationary, the  $f$  waves sent out in one second towards the observer  $O$ , occupy a distance  $V$  and the wavelength will be  $\frac{V}{f}$ .

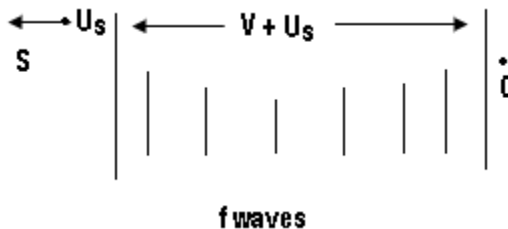
(2)



### Source moving towards the stationary observer:

- If the source S moves with a velocity  $U_s$  towards the observer O, the  $f$  waves sent out will occupy a distance  $(V - U_s)$ , since S has moved a distance of  $U_s$  towards O in one second.
- The wavelength  $\lambda^1$  of the waves reaching O is now  $\frac{(V - U_s)}{f}$ .
- The apparent frequency  $= \frac{V}{\lambda^1} = \frac{V}{(V - U_s)/f} \Rightarrow f^1 = \frac{V}{V - U_s} f$ .
- Since  $(V - U_s)$  is less than  $V$ ,  $f^1$  is greater than  $f$ , and the apparent frequency appear to increase.

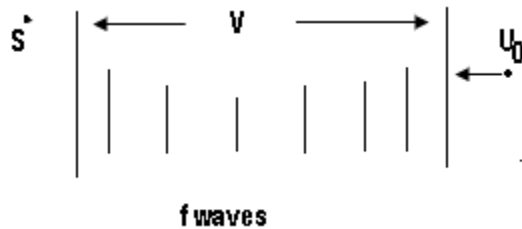
(3)



### Source moving away from the stationary observer:

- In this case, the  $f$  waves sent out in one second occupy a distance  $(V + U_s)$ .
- The wavelength  $\lambda^1$  of the waves reaching O is thus  $\frac{(V + U_s)}{f}$ , and the apparent frequency  $f^1$  is given by  $f^1 = \frac{V}{\lambda^1} = \frac{V}{\frac{(V + U_s)}{f}} \Rightarrow f^1 = \frac{V}{V + U_s} f$ .
- Since  $(V + U_s)$  is greater than  $V$ ,  $f^1$  is less than  $f$ , and hence the apparent frequency decreases, when the source moves away from the observer.

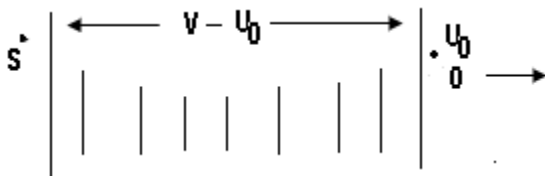
(4)



### Source stationary with the observer moving towards it:

- Since the source is stationary, the  $f$  waves sent out by the source  $S$  towards the moving observer  $O$ , occupies a distance  $V$ .
  - The wavelength of the waves reaching  $O$  is  $\frac{V}{f}$ , and unlike in the other two previous cases, the wavelength is unaltered.
  - The velocity of the sound wave relative to  $O$  is not  $V$ , and  $O$  is moving relative to the source.
  - The velocity of the sound wave relative to  $O$  is given by  $(V+U_0)$  in this case, and hence the apparent frequency  $f^1$  is given by  $f^1 = \frac{V+U_0}{\frac{V}{f}}$
- $$\Rightarrow f^1 = \frac{V+U_0}{V} f.$$
- Since  $(V + U_0)$  is greater than  $V$ ,  $f^1$  is greater than  $f$ , and as such the apparent frequency is increased.

(5)



### **Source stationary, and observer moving away from it:**

- In this case, the wavelength of the waves reaching O is unaltered, and is given by  $\frac{v}{f}$ .
- The velocity of the sound waves relative to O =  $v - u_0$ , and hence the apparent frequency,  $f^1 = \frac{v - u_0}{\frac{v}{f}}$   
 $\Rightarrow f^1 = \frac{v - u_0}{v} f$ .
- Since  $(v - u_0)$  is less than  $v$ , the apparent frequency  $f^1$  appears to be decreased.

### **(6) The source and the observer are both moving:**

- If the source and the observer are both moving, the apparent frequency  $f^1$  is given by  $f^1 = \frac{v^1}{\lambda_1}$ , where  $v^1$  = the velocity of the sound waves, relative to the observer.

$\lambda_1$  = the wavelength of the waves reaching the observer.

(Q1)(a) What is Doppler effect?

(b) Explain why Doppler effect occurs.

Soln:

(a) This is the apparent change in the frequency or the wavelength of a source of sound or light, due to the relative motion between the observer and the sound or the light.

(b)- When a source approaches a stationary observer, the observer receives the wavefront which is relatively more crowded, than if the source was stationary.

- The wavelength of the wave is therefore shorter, and the frequency (pitch) appears to rise.

- On the other hand, when the source recedes or moves away from the observer, the observer receives the wavefront which are further apart than if the source is stationary.

Therefore the wavelength of the wave he receives is larger and he receives fewer waves per second, i.e. the apparent frequency is low.

(Q2) A man moves towards a stationary source of sound, emitting note of frequency 200Hz. If the man's motion is such that in 2 seconds he moves through a distance of 16m, calculate the frequency of the note heard by the man. [Take the velocity of sound in air =  $330\text{ms}^{-1}$ ].

N/B:

In this case, the source is stationary and the observer is moving towards it. We therefore use the formula  $f^1 = \frac{V + U_0}{V} f$ , where  $f^1$  = the apparent frequency or the frequency heard by the man.

$U_0$  = the velocity of the man or the observer.

$V$  = the velocity of sound in air.

$f$  = the frequency of the source.

Soln:

$V = 330\text{ms}^{-1}$ ,  $f = 200\text{Hz}$  and  $U_0 = ?$ .

The man's velocity  $U_0 = \frac{\text{distance}}{\text{time}} = \frac{16}{2} = 8\text{m/s}$ .

$$f^1 = \frac{V + U_0}{V} f = \frac{(330+8)}{330} \times 200$$

= 205Hz.